Simulation of Oxygen Penetration and Decarbonisation in EAF Using Supersonic Injection System

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1. Introduction

There are several ways in the literature to estimate the depth reached by a supersonic O2 flow in liquid steel. They can be summarized in three methods: energetic, dynamic and numerical methods. Energetic methods are based on an energetic balance between the momentum of the jet and the energy needed for the penetration. The dynamic methods are based on the evaluation of the impact force of the jet upon the surface of the liquid steel, which requires the inclusion of the features of the nozzle and of the jet:

\[ F = \rho_0 v_0^2 \frac{d^2}{4} \]

where \( v_0 \) is the jet velocity at the nozzle outlet. If the volume of displaced liquid is constant and known, the depth reached by a vertical jet in a steel bath is given by:

\[ D = \frac{2F}{\pi \rho \rho_{\text{steel}} b H^2} \]

where \( H \) is the height of the nozzle above the steel, \( g \) is the gravity constant, \( \rho \) is the density. The problem with the use of this kind of method is the value of the proportionality constant \( \beta \) that must be determined experimentally through hydro-dynamical trials, which not always offer reliable and significant results for the industrial practice. The numerical methods are based essentially on a finite element or finite difference calculation (i.e., CFD software). There are several problems bonded to the convergence of the solutions for high velocity jets. Problems are generated by the numeric definition of an interface surface and by the formation of steel drops in the jet itself. Moreover, as these techniques are very time consuming, the use for an on-line control can find a significant obstacle.

To overcome these problems, an update of the energetic approach has been developed in this study. Energetic approach has been integrated by the principle of the dynamic approach for the approximation of the volume of the surrounding by the liquid phase.

The model elaborated for the injection of a supersonic jet can lead to a computation about decarburisation phenomena through the determination of the computed liquid steel/injected O2 interface.

A thermodynamic and kinetic model of the bath during the refining period has been built. Its main aim is to provide a fast tool to estimate the effect of oxygen and carbon injection and to optimise the process in EAF.
The supersonic injection of O₂ has been performed by an improved shape of the nozzle featuring the KT Supersonic Injection System developed by Techint.

The validation has been performed on an industrial EAF plant equipped by a burning injection system of three fixed lances endowed with special nozzle profile. The EAF is also using a conventional door lance.

2. The Theoretical Background

2.1. About the Laval Nozzle

Here are reported Laval nozzle classical isentropic formulas (from Hugoniot):

\[
\frac{dA}{A} = (M^2 - 1) \frac{dv}{v} \quad \text{...........................}(3)
\]

\[
\frac{dA}{A} = \frac{1 - M^2}{\gamma M^2} \frac{dP}{P} \quad \text{...........................}(4)
\]

\[
\frac{dP}{P} = M^2 \frac{dv}{v} \quad \text{...........................}(5)
\]

\[
\rho v A = \text{const} \quad \text{...........................}(6)
\]

These relations are submitted to: short conduct with small curvature and divergence, perfect O₂, negligible stresses due to viscosity and negligible energy loss and heat exchange (adiabatic flow). These equations show the outlet velocity of the jet given the characteristics of the nozzle and of the O₂ inlet.

Once the geometry of the nozzle is fixed, if the environment temperature is assumed as constant: outlet velocity, inlet pressure and O₂ flow are mutually related by:

\[
Q = A_{\text{throat}} \left[ \frac{2 P_{\text{throat}}}{\gamma - 1} \left( 1 - \left( \frac{P_{\text{throat}}}{P_{\text{in}}} \right)^{\frac{\gamma - 1}{\gamma}} \right) \left( \frac{P_{\text{throat}}}{P_{\text{in}}} \right)^{\frac{1}{\gamma}} \right] \quad \text{...........................}(7)
\]

2.2. About Coherence Length

Supersonic O₂ jet behaviour has been studied extensively: O₂ exiting the nozzle meets the outside air creating a turbulent mixing and transport region. This process involves the jet diameter increase and the decrease of jet velocity while rising distance from the nozzle. The coherence region is the region nearby the nozzle where the axial velocity and the Mach number are constants and equal to the value at the nozzle exit. Outside this region, axial velocity becomes subsonic (developed lateral flux). According to Thiring and Newby the coherence length is:

\[
\frac{d^*}{d} = 4.2 + 1.1 \left( M^2 + 1 - \frac{T_j}{T_a} \right) \quad \text{...........................}(8)
\]

This formula takes the density of the surrounding medium (\(\rho_{\text{ambient}}\)) into account with the term \(d^*\):

\[
d^* = \left( \frac{\rho_{\text{ambient}}}{\rho_{\text{O}_2}} \right)^{0.5} d_0 \quad \text{...........................}(9)
\]

Increasing the pressure of the jet the Mach number and the geometry of the nozzle change and these lead to the change of the coherent jet length.

In the same way, the increase of environment temperature and of the flow rate of the jet, and the decrease of jet temperature cause the increase of the coherent length.

3. The Theoretical Model

3.1. Effect of the Liquid Steel Surface Tension

It is considered a spherical cavity generated by the jet in the steel (radius \(R_0\)); the energy balance is:

\[
\frac{1}{2} \rho_{\text{steel}} v^2(x) = \rho_{\text{steel}} \gamma D + \frac{2 \sigma}{R_0} \quad \text{...........................}(10)
\]

where \(D\) is the depth of the cavity in the molten bath (Fig. 3).

Under the hypothesis of a developed flow of O₂ the axial velocity is given by the equation:
where $x$ is the coordinate coaxial with the axis of the lance and starts from the outlet of the nozzle, $k$ is constant for a given temperature and pressure of injected O$_2$ and a given conditions (i.e. pressure, temperature, chemical composition) of the surrounding air. This is a good approximation, but for a more precise calculation one should consider the “jet pole” of the flow as suggested by the theory of developed jets.

A standard value for the surface tension is assumed:

$$\sigma = 1.5 \text{ J m}^{-2}$$

The radius of the cavity $R_0$ can be approximated as:

$$R_0 = (He) \sin \phi$$

Where the angle $\phi$ has a standard value of 9° for a developed jet of air in air. The balance equation is a third order algebraic equation whose real solution for $He=0.4$ m is:

$$D = 11.91 \text{ cm}$$

Ignoring the influence of the surface tension, with the same input data the solution is:

$$D = 11.95 \text{ cm}$$

These results demonstrate that the surface tension has a negligible influence on the depth of penetration in liquid steel. With a supersonic coherent jet this small difference vanishes, thus this hypothesis is more confirmed. Moreover, the computation performed by the energetic approach described in Eq. (10) suggests that the sphere shape is not the most suitable one for the description of the O$_2$ jet impinging the steel. The surface of a paraboloid of revolution can represent a more reliable description of the interface shape.

### 3.2. Axial Velocity Calculation for an Inclined Jet over a Steel Bath

The axial velocity for an inclined nozzle placed in the walls of the furnace can be calculated through the use of the concepts exposed previously. A supersonic jet can be divided in two main regions (Fig. 4): the coherent region, featured by a potential core and a thin turbulent mixing layer, which is followed by the zone of transition toward the completely developed flow that characterizes the incoherent region.

While the axial velocity in the first region can be considered as constant, in the second one follows the law given by the Eq. (11). For these reasons the velocity of O$_2$ along the axis of symmetry of the jet during the passage through the air is:

$$v_{axial, air}(x) = \begin{cases} v_0 & x < L_p \\ \frac{k_0 v_0 D_p}{x} & x \geq L_p \end{cases}$$

This situation verifies when the nozzle is positioned above the slag.

The profile of the velocity which features the several transverse sections which compose the O$_2$ jet impinging the steel can be computed on the basis of a Gaussian distribution of the velocity whose maximum is centered along the symmetry axis of the jet (Appendix A). The jet can meet the slag in two different ways: as a coherent flow or as a developed flow. The theory developed for the surrounding air can be applied also in the case of the jet flowing through the slag. As a matter of fact the Thiring formula (8) can be
used to estimate the length of the coherent flow in slag. The calculation is simple if the nozzle area is surrounded by the slag (this is the case encountered during the refining period with a foamy slag). In case the jet loses its coherence in the impact on the steel bath (this hypothesis is justified by calculating the Eq. (8) for an oxygen jet in liquid steel) the axial velocity can be estimated through the energy balance in a generic point of the axis inside the bath:

\[ \frac{1}{2} \rho_{0} v^2(x) = \frac{1}{2} \rho_{0} \left[ k_{steel} v_{steel}(x_{steel}) \right] x' \sin \alpha - \rho_{steel} g \left[ \rho_{steel} \right] (x) \]

\[.............................(15)\]

The maximum vertical depth (from the surface) reached by the flow is calculated at the point where the axial velocity goes to zero (\(x_{\text{max}}\)):

\[ D = (x_{\text{max}} - x_{\text{steel}}) \sin \alpha \]

\[.............................(16)\]

Through this model the velocity profile of the O₂ jet is completed and plotted (Fig. 5).

### 3.3. The Calculation of the Interface Surface

In order to avoid the use of a CFD software or a statistic analysis the interface surface between the O₂ jet and the steel bath has been assumed schematically not as a sphere, but as a revolution paraboloid whose axis is the same of the jet one. This representation seems to be more realistic for a coherent supersonic jet, because it can permit to simulate a jet in which the penetration depth is larger than the radius of its transversal section. In this scheme the free surface of the bath will be represented as a plane surface. This surface is simple to be handled mathematically and it can be determined as a function of the main parameters involved in the injection practice. If the jet is not perpendicular to the bath, then the paraboloid will have a certain angle \(\alpha\) measured from the plane to its axis as shown below (Fig. 6).

In order to evaluate the equation of these surfaces, they are represented in a three dimensional Cartesian space (\(x, y, z\)) (Fig. 6). The general equation for a paraboloid is:

\[ \frac{x^2}{H} + \frac{y^2}{H} = 2p z \]

\[.............................(17)\]

Where the constant \(p\) can be determined imposing two physical conditions: the depth reached by the jet into the steel bath (previously calculated) and the volume of the parabolic section inside the bath, equal to the fluid displaced by the flow (provided the impulsive force generated by the O₂ jet):

\[ \frac{V}{\rho_{steel}} = \frac{1}{2} \rho_{0} v_{0}^2 \left[ \rho_{steel} \right] \]

\[.............................(18)\]

Provided the depth of the jet penetration into the bath, the shape of the paraboloid is determined by imposing that the geometrical volume of the paraboloid is equal to the one occupied by the impinging jet. Since the paraboloid is inclined, the surface has been determined by an iterative procedure (Appendix B). Once known the value of the parameter \(p\), the surface of interface can be determined by integrating the Eq. (17).

\[ S = \int_{\Gamma} dS \]

\[.............................(19)\]

Where \(\Gamma\) is the integration dominium. Moreover, the intersection between paraboloid and plane of the bath can give an approximate measure of the impinging region.

### 3.4. Oxidation and Decarburization

Assumed that the refining period—of an EAF—is the time starting from the complete melting of the metallic charge until tapping, during this period the steel inside the furnace may be considered a liquid phase. The injection of oxygen in the bath produces two main effects: the oxidation of the elements in solution with the steel, and the generation of heat that contributes to consequent electrical energy savings. Both these phenomena have to be described by a single model, since they are closely related.

The energy developed by the oxidation is valuable by the reaction enthalpy, once the number of reacting moles are known.
Then the problem is to choose the chemical species oxidized in the bath and the calculation of the number of moles of a certain chemical species in solution that are oxidized by oxygen during a fixed time range.

The first point can be solved through the use of oxygen potentials.

Considering an oxidation reaction:

\[ p[\text{metal}] = O_2 \overset{\Sigma_f}{\longrightarrow} (\text{oxide}) \] ..........................(20)

According to Wagner theory the oxygen potential for this reaction is:

\[ \Delta \mu_{O_2} = \Delta \mu_{\Sigma_f} - pRT \ln(a_{\text{metal}}) + fRT \ln(a_{\text{oxide}}) \] ..........................(21)

Since thermodynamic states that the element with the lowest oxygen potential is the first to be oxidized, then the problem of the choice of the chemical species to be oxidized is solved for a certain instant. The values of \( \Delta \mu^* \) for oxidation reactions can be evaluated as:

\[ \Delta \mu^* = A + B \cdot T \log T + C \cdot T \] ..........................(22)

See Table 1 for the values of \( A, B \) and \( C \) used. Phosphorus has not been considered, because the used scrap has a low phosphorus content.

To quantify the moles reacting, kinetics of the bath must be taken into account.

The diffusion laws give the number of the moles for a certain species that can move to the interface surface with the jet in \( \Delta t \). In terms of concentrations (\( \xi \)),

\[ \xi(t, \Delta t) = \xi(t) \beta_S \Delta t \] ..........................(23)

where the transport constant \( \beta \) for each species can be calculated in turbulent conditions as:

\[ \beta = \sqrt{D^2} \rho \sigma^{-1/2} \] ..........................(24)

Since the difference between the diffusion constants for the elements dissolved in the steel bath are not significant for the description of the treated industrial case, only a single value for \( D \) has been used for all the species to simplify the computation:

\[ D = 0.5 \cdot 10^{-9} \text{ m}^2 \text{ s}^{-1} \] ..........................(25)

The velocity \( u_t \) is the average value of the steel velocity in correspondence of the cavity generated by the \( O_2 \) flow inside the bath. The velocity of steel can be calculated applying the momentum transfer between jet and bath:

\[ u_{\text{steel}}(x) = \frac{1}{\rho_{\text{steel}}} \int_{x_{\text{min}}}^{x_{\text{max}}} v(x) \, dx \] ..........................(26)

and \( u_{\text{metal}} \) is the average value over the range covered by the jet inside the bath:

\[ u_{\text{metal}} = \frac{1}{\rho_{\text{metal}}} \int_{x_{\text{min}}}^{x_{\text{max}}} v_0(x) \, dx \] ..........................(27)

### 3.5. The Carbon Injection and Its Integration in the Model

In the common melt shop practise and in the observed heats during the refining period of the melting operation, the carbon injection in the slag takes place too.

This is needed for some main reasons: first of all to avoid an excessive oxidation of the slag, to increase scrap yield, to promote the formation of foamy slag and to protect the furnace walls from the direct exposition to the electric arc.

The role played by the carbon injection in the decarburisation phenomena has been studied in the same way operated for the oxygen injection through the use of kinetic factors \( \xi \) (each one different for each component of the slag):

\[ \Delta \xi = \xi \Delta t \] ..........................(28)

and with the introduction of carbon potentials (analogous to the oxygen ones) for the reduction reactions in the slag:

\[ \frac{1}{n} (\text{MeO}_n) + C \overset{2}{\longrightarrow} \frac{2}{n} \text{[Me]} + \text{CO} \uparrow \] ..........................(29)

The reactions of SiO\(_2\) and Al\(_2\)O\(_3\) were neglected because their reduction potential is too high in the EAF process.

The moles of each species in the bath is completed by adding the contribute of the reduction from the slag and subtracting the oxidised moles of each species. In this way, combining in the same interval the effect of the two phenomena of the oxygen and carbon injection, the model can estimate the composition of slag and steel bath (and the equilibrium between them) dynamically during the whole refining period. The total chemical energy developed is calculated by adding the enthalpies of oxidation for each oxidation reaction in an interval, and subtracting the reduction ones in the slag. The oxygen activity in the bath is constantly updated considering the equilibrium with the FeO in the slag:

\[ \frac{[\text{O}_2]}{[\text{FeO}]} = \frac{6.320}{T} + 2.765 \] ..........................(30)

### 4. Implementation of the Model

If the refining time is divided in several intervals, in each of them a thermodynamic description of the interface for each oxidation reaction can be performed through the use of the potential of oxygen.

Then, since the \( O_2 \) flow is known by imposing the technologic parameters of the process, the number of moles of oxidized elements can be evaluated starting from the one with the lowest potential and proceeding towards the higher potential ones. Only the free moles of oxygen left by each oxidation can react with the elements with higher potentials.

As a matter of fact from Eq. (24) the amount of each species coming to the surface can be estimated. If there is oxygen left from the oxidation of the first element featured
by the lowest oxygen potential it becomes available for the oxidation of the next lower element in the oxygen potential scales and so on.

On every time interval the activities of the elements at the surface can be evaluated considering the moles consumed in the former time step by the oxidation reactions and the diffusion laws. The temperature is evaluated at every time step by adding the heat developed by each reaction in terms of temperature difference on steel. This is true under the adiabatic hypothesis for the reaction surface:

$$
\Delta E_r(J) = H_r(J \text{ mol}^{-1})[m(t) - m(t + \Delta t)](\text{mol}) \ldots (31)
$$

The enthalpy can be evaluated by:

$$
\Delta H_r = H^0_r + c_p \left[ T - 298.15 \right] \ldots \ldots \ldots \ldots \ldots (32)
$$

where $H^0_r$ is the value of enthalpy at standard conditions for the $j$-th reaction and $c_p$ is the specific heat at constant pressure. They can be easily evaluated according to Barin–Knacke data.

The sequence of this computation is developed for each time step taken into account to follow the thermo-chemical evolution of the considered system.

The decarburisation process can be limited by the flow of oxygen and by the carbon diffusion. When the oxygen consumes all the carbon available on the reaction surface and can oxidize also elements with higher potential, decarburisation is limited by the carbon diffusion to the interface. On the other hand, when the carbon consumes all the oxygen at the interface, the decarburization is controlled by the oxygen flow.

The application of the model in the condition of the observed industrial process shows that this transition takes place in the range of 0.2–0.4% (weight) of carbon in the bath. This matches the data presented in the bibliography about this topic.

The same can be observed for the other elements considered in the model.

5. Experimental Procedure and Results

The model exposed in this article has been implemented in dedicated software. In order to use it, it must be validated (the kinetic constants must be tuned) on a real EAF. The calibration has been done on the TenarisDalmine furnace and the original software has been modified in order to simulate the different types of lances present in the plant. The software has been calibrated over 100 heats and it has been checked on other hundreds heats.

Temperature, oxygen activity in the steel and the composition of slag and steel have been monitored through a series of sampling trials performed during the refining period of each heat.

Through these experiments the estimation of the kinetic constants has been possible and the results are shown in Table 2.

Once those constants have been obtained, the model has been tested over 100 heats (Figs. 8 and 9).

The experimental curve has been obtained through the sampling of liquid steel in different steps of the refining period and their analysis through spectroscopic analysis or SEM spectrometer.

Table 2. Kinetic constants obtained by the software calibration.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oxidation reaction</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>FeO Reduction</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>Cr$_2$O$_3$ Reduction</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>MnO Reduction</td>
<td>0.001</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 8. Values predicted by the software vs. values measured for carbon.

Fig. 9. Values predicted by the software vs. values measured for FeO in slag.
Since the model describes the trend of the concentrations in the bath and in the slag also the output curves of the other elements can be compared to the experimental ones, here are shown only the most important ones, and they offer a very reliable results.

6. Conclusions

The decarburisation model described in this article can evaluate dynamic wise the composition of the bath and slag during the refining period of the melting operation within the EAF. This calculation mainly depends on the characteristics and the technology of the injection system.

For this reason it must be calibrated (for what concerns only the chemical part since the penetration model is not influenced by the kinetics of the chemical species in the bath) in order to define the kinetic constants of the process.

This means that the physical part about the impact of the oxygen jet inside the steel bath can be used for any plant without being modified, while for its chemical effects one must know the values of the kinetic constants. For a rapid calculation, with less precision, values obtained for analogous cases can be applied.

Once calibrated, the model gives the dynamical evolution of the composition of the bath during the refining period for every element implemented with this method (except phosphorus for the reasons mentioned above). Through the study of the reaction equilibria also the slag composition evolution can be evaluated.

This is very important for the refractory life and for the creation of a plant procedure that can prevent its wear.

The calibrated model can estimate the effects of any change of the operative parameters: the positioning of the burning-injection lances in the furnace, the pressure and change of the operative parameters: the positioning of the burning-injection lances in the furnace, the pressure and evolution can be evaluated.

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This is very important for the refractory life and for the creation of a plant procedure that can prevent its wear.

The calibrated model can estimate the effects of any change of the operative parameters: the positioning of the burning-injection lances in the furnace, the pressure and temperature of the O$_2$ flow, a different nozzle design.

At present time it has been used and validated on the basis of the decarburisation improvements brought by the new injection technology equipped by an improved nozzle design developed by Techint Technologies and named KT Injection System. This system is often applied in a multi-nozzle configuration which can assure a better homogenisation of the steel during the decarburisation process.

Nomenclature

- $\gamma$: Specific heat ratio of a gas
- $\sigma$: Surface tension (J m$^{-2}$)
- $\alpha$: Angle of the jet (deg)
- $\beta$: Transport coefficient (m/s)
- $\xi$: Proportional kinetic constant for bath
- $\xi_i$: Concentration of moles of the specie $i$ (mol m$^{-3}$)
- $\Delta m$: Concentration of an oxide in the slag (mol m$^{-3}$)
- $\Delta m_{st}$: Oxygen standard potential (kJ/mol)
- $\rho_{miso}$: Density of the medium where the jet flows (kg/m$^3$)
- $\rho_{o}$: Jet density (kg/m$^3$)
- $\rho_{a}$: Proportional kinetic constant for oxide $i$ slag (s$^{-1}$)
- $\rho_{o}$: Density of the injected gas at the entry transversal section of the nozzle (kg/m$^3$)
- $\rho_{miso}$: Density of the liquid steel (kg/m$^3$)
- $\Delta t$: Time interval (s)
- $\xi_i$: Concentration of an element in the steel (mol/m$^3$)
- $A$: Transversal section of the nozzle (m$^2$)
- $A_{th}$: The smallest transversal section of the nozzle (m$^2$)
- $a$: Activity
- $C_{p, st}$: Calorific power of the steel bath (J kg$^{-1}$ K$^{-1}$)
- $D$: Depth of the cavity in the steel (m)
- $D_{i}$: Diffusion constant in liquid steel for a given species $i$ (m$^2$/s)
- $E$: Energy (J)
- $g$: Gravity acceleration (m/s$^2$)
- $h_{miso}$: Vertical depth in the bath calculated at a certain abscissa (m)
- $H$: Enthalpy (J mol$^{-1}$)
- $H_e$: Height of the exit transversal section of the nozzle above the plane level of the bath (m)
- $\Delta H_{st}$: Enthalpy at standard conditions (J mol$^{-1}$)
- $k$: Constant of developed flow (for a given gas and a given medium)
- $k_{miso}$: Constant for a developed flow of oxygen in steel (air)
- $L_e$: Coherence length (m)
- $M$: Mach number during the iso-entropic expansion of the injected gas
- $m_i$: Number of moles (mol)
- $m_{Fe}$: Moles of Fe in the bath (mol)
- $P$: Pressure (Pa)
- $P_{miso}$: Pressure at the entry section of the nozzle (Pa)
- $P_{th}$: Pressure at the smallest section of the nozzle (Pa)
- $Q$: Gas flow rate (m$^3$/s)
- $R$: Perfect gas universal constant (J mol$^{-1}$ K$^{-1}$)
- $R_{c}$: Radius of the cavity formed in the liquid steel (m)
- $r$: Radius of the cylinder-Region 2 (Appendix B) (m)
- $R_{o}$: Radius of the outlet section of the nozzle (m)
- $S$: Interface between the gas flow and the steel bath (m$^2$)
- $T$: Absolute temperature (K)
- $T_{l}$: Absolute temperature of the injected gas during the iso-entropic expansion within the nozzle (K)
- $u_{av}$: Average velocity of steel in correspondence of the jet surface (m/s)
- $v$: Velocity (m/s)
- $V$: Volume of the cavity formed by the gas flow in the steel (m$^3$)
- $V_{l}$: Velocity of the flow at nozzle outlet (m/s)
- $v_{a}$: Axial velocity (component of the velocity along the axis of symmetry of the jet) (m/s)
- $v_{axial,av}$: Velocity of the injected gas along the axis of symmetry of the jet during the passage through the surrounding air (m/s)
- $v_{sag}$: Axial velocity of the gas after the passage through the slag layer (m/s)
- $W$: Weight of a mole of Fe (kg/mol)
- $x$: Axial coordinate taken from the nozzle outlet (m)
\[ v(x, \zeta) = v_{\text{ax}}(x) \exp(\varphi \zeta) \] ............................(A-1)

where the constant \( \varphi \) shall be determined case by case in function of the medium and the condition of the atmosphere inside the furnace.

Appendix B. Geometrical Volume Calculation

Under the hypothesis of a paraboloid shape, the cavity can be divided into three regions (Fig. A-1).

The region 1 volume is:

\[ V_1 = \frac{\pi r^4}{p^2} \] .............................(B-1)

where the meaning of \( r \) is clearly expressed in the drawing (Fig. A-1).

For region 2:

\[ V_2 = \int_0^\pi \int_0^\infty \int_0^{\cot(\alpha)} \rho d\varphi \rho d\theta \] .............................(B-2)

where \( \Omega_2 \), the integration dominium, is the projection of the outlined surface in Fig. A-1 on the \( \tilde{x}-\tilde{y} \) plane.

For region 3:

\[ V_3 = \int_{\tilde{D}} \rho dV \] .............................(B-3)

where \( \Omega_3 \), the integration dominium, is the projection of the region 3 (Fig. A-1) on the \( \tilde{x}-\tilde{y} \) plane. This value can be obtained through numerical calculations.

The sum \( V_1 + V_2 + V_3 \) gives the total geometrical volume of the cavity and it is a function of \( p \) and of \( r \). Their value can be obtained imposing the depth reached by the jet previously calculated (16) and the physical volume of moved liquid (18).

Once known the constant \( p \), the dominium \( \Gamma' \) (19) can be easily determined intersecting the paraboloid (17) with the steel bath plane:

\[ \tilde{z} = \frac{D}{\sin(\alpha)} + \tilde{x} \cot(\alpha) \] .............................(B-4)